Reply to "Comment on 'Extended self-similarity in turbulent flows'"

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In this Reply we question the conclusion of van de Water and Herweijer (WH) [preceding Comment, Phys. Rev. E **51**, 2669 (1995)] about the evidence of multiscaling behavior in the dissipation range of turbulence. We perform the same analysis suggested by WH for the data set used by Benzi *et al.* [Phys. Rev. E **48**, 29 (1993)] to establish extended self-similarity. At variance with WH, we do not observe any evidence of multiscaling. We argue that data filtering in WH could produce a misleading effect at very small scales. The combined effect of multiscaling and extended self-similarity is an important question that needs to be investigated in more detail, both theoretically and experimentally.

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In their Comment (Ref. [1], hereafter denoted WH) to the paper of Benzi *et al.* [2], WH claim that extended self-similarity cannot be observed in turbulence because of multiscaling effect. The WH argument can be rephrased in this way: let us suppose that without correction due to multiscaling, the structure function of the velocity difference can be written as

$$G_n(r) \propto U_0^n \left(\frac{r}{L} f(r/\eta)\right)^{\zeta(n)}, \qquad (1)$$

where $G_n(r) = \langle [\delta V(r)]^n \rangle$. Then the point raised by WH is: even if (1) is true, we should observe small but significant deviations induced by multiscaling. Thus, $f(r/\eta)$ is no more a universal function independent of n and Eq. (1) should be rewritten as

$$G_n(r) \propto U_0^n \left(\frac{r}{L} f_n(r/\eta)\right)^{\zeta(n)}. \qquad (2)$$

WH support their claim by plotting $\frac{f_n}{f_2}$ as a function of $\frac{r}{\eta}$; see Fig. 1 of WH. Although it is theoretically interesting, the claim raised by WH cannot be justified by the experimental results, as we will show in this Reply.

First, let us make some theoretical considerations regarding multiscaling. The scale below which one may eventually observe multiscaling behavior is not fixed by theory. If one thinks that multiscaling starts just at the end of the inertial range, as WH seem to believe, then, by plotting G_p against G_2, one should observe that for a scale smaller than those of the inertial range, the local slope $\frac{d\log_{10}(G_p)}{d\log_{10}(G_2)}$ should decrease for $p > 2$. This is due to the fact that the range of scaling of $G_p(r)$ should increase for increasing values of p, as can easily be deduced from the theoretical argument given by Frisch and Vergassola [3]. There is experimental evidence, both in Benzi and co-workers [2], [4] and in Stolovitsky and Sreenivasan [5], that this is *not* the case. Thus, we reach the conclusion that the experimental results discussed in Refs. [2] and [4] are not affected by multiscaling. Due to the fact that the flow geometry is the same for Refs. [1], [2], and [4], one may wonder why multiscaling is observed only in [1]. Let us remark that the difference in Reynolds's number between the two sets of experiments is not large enough to justify, from the multiscaling point of view, the difference claimed by WH.

In order to address this matter, one must discuss whether the deviation to the extended self-similarity observed by WH is well outside experimental error. As we shall see, this is not the case. The quantity discussed by WH is

$$\ln\left(\frac{f_p(r)}{f_2(r)}\right) = \frac{1}{\zeta(p)}\ln G_p(r) - \frac{1}{\zeta(2)}\ln G_2(r). \qquad (3)$$

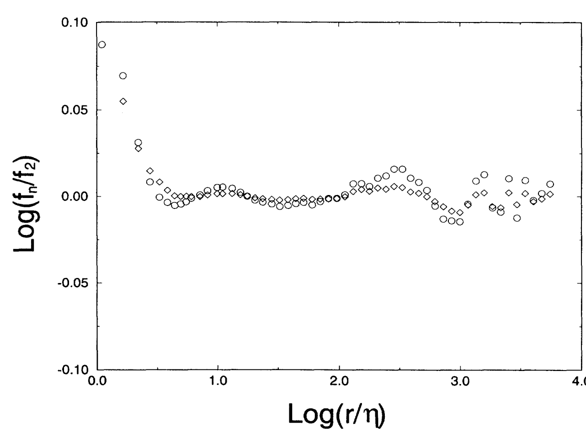

FIG. 1. The $\ln(f_4/f_2)$ (◇) and of $\ln(f_6/f_2)$ (○) are reported as a function of r/η. The f_n have been obtained from the structure functions computed from measurements of turbulence behind a cylinder at $R_\lambda = 140$. The probe was at 30 cylinder diameter downstream and the wind speed was $U_0 = 1.5$ m/s. The hot wire was 25 μm thick and 0.25 mm long; thus, it allows us to resolve the Kolmogorov scale $\eta = 0.5$ mm. The hot wire signal filtered at $U_0/\eta = 3$ KHz has been linearized to get the velocity.

This quantity can be computed only if we know $\zeta(p)$ and $\zeta(2)$. Even using extended self-similarity (which is not justified in this case) the accuracy of these numbers is of order 0.01, 0.02 (which is quite good!). Typical numbers are $\zeta(2) = 0.695$–0.705, $\zeta(4) = 1.27$–1.29, and $\zeta(8) = 2.21$–2.25. Getting all the numbers together we obtain that $\ln\left(\frac{f_p(r)}{f_2(r)}\right)$ is known with an accuracy of order 0.05 in the dissipation range. This tells us that significant deviations are those greater then 0.05. Deviations greater than 0.05 are observed only for scales smaller than or equal to 10 η. However, in the experiment of WH, there is another important source of error for scale near or below 10 η. According to WH, the experimental data are filtered at 10 kH with a 4 pole filter (at least 24 db/octave). Now in the case of a jet the average wind speed is about 10 m/sec and the Kolmogorov length is 10^{-4} m. This corresponds to a frequency of about 10^5 kH, i.e., 10 times bigger than what is needed to resolve the Kolmogorov scale. Thus, in principle, data below 10 η are reproducing the effect of the filtering. If WH have used a deconvolution from the filtered data, we believe that they have introduced an error which could be estimated roughly on the order of several percent. This error should be added to the error discussed above. Let us remark that the bending of the lines presented in WH is quite strong just at the filter response. The situation is only marginally improved in the grid turbulence simply because R_λ is about two times smaller than in the turbulence produced by the jet. Therefore, we can suggest that WH probably want to present evidence of multiscaling inside an error bar which is of the same order of the evidence itself.

In order to support the previous analysis, we show in Fig. 1 the quantities $\ln\left(\frac{f_n(r)}{f_2(r)}\right)$ for $n = 1, 2, 4, 5, 6$ obtained by the data discussed in Ref. [2] at Reynolds's number of about 9000. As one can see, there is no deviation up to scales smaller than 5 η. Let us also mention that the same result is observed in direct numerical computation; see Ref. [6].

The results in Fig. 1 clearly show that the WH analysis is too much affected by the filter response at small scales to make any significant claim on multiscaling. Finally, the question concerning the scaling of $\langle|\delta V^3(r)|\rangle$ has been answered in Ref. [4], where it has been shown that $\langle|\delta V^3(r)|\rangle \approx |\langle\delta V^3(r)\rangle|$ with very high accuracy.

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